UC SAN DIEGO, DEPARTMENT OF MATHEMATICS

MATH281A Homework 1

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Exercise 1.1.2.

Hint: Use Lagrangian multipliers. The basic idea is that since it is iid then everything should be symmetry.

Exercise 1.1.3.

Pay attention the α_is in (a) are fixed. So they are not the variables you are going to optimize. Lagrangian multipliers will work, but if you define $Y_i = \sqrt{\alpha_i} X_i$, things will be better.
For (b), just use

$$var(\sum_i \alpha_i X_i) = \sum_i \alpha_i^2 var(X_i) + \sum_{i \neq j} \alpha_i \alpha_j cov(X_i, X_j)$$

Exercise 1.1.12.

Go direct integration and the answer will come out. To show the finite moment you may need to control the tail with inequality

$$\frac{1}{(1+|x|)^k} < \frac{1}{|x|^k}$$

Exercise 1.4.1.

Hint: Use the cdf and do a transformation then you will find what you need.

Exercise 1.4.2.

Hint: Cdf still works. For instance:

$$\mathbb{P}(-\log X < x) = \mathbb{P}(X < e^{-x})$$

Then compare the result and the cdf of an exponentially distributed variable.

Exercise 1.4.13b.

The same with the previous questions. Check Weibull distribution and its cdf in Wikipedia.

Exercise class.1.

Hint: Use equality

$$\sum_i (X_i - \bar{X})^2 = \sum_i X_i^2 - n(\bar{X})^2$$

and note that $\mathbb{E}(X_i^2) = var(X_i) + (\mathbb{E}(X))^2$. Do the same trick to find $\mathbb{E}(\bar{X})^2$.

Exercise class.2.

This equation may make life easier:

$$\sum_i (X_i - \bar{X})^2 = \sum_{\{i,j\}} \frac{1}{n}(X_i - X_j)^2$$

The right hand side runs all combination of $\{i, j\}$. First prove the equation above, then carefully analyze your target using the formula for variances.

Exercise class.3.

For skewness, try to prove that for iid sample $X_1, \ldots, X_n$, we have

$$\mu_3(\sum_i X_i) = n\mu_3(X_i)$$

Here μ_3 stands for third central moment.
For kurtosis, try to show that for iid sample $X_1, \ldots, X_n$, we have

$$\mu_4(\sum_i X_i) - 3(\mu_2(\sum_i X_i))^2 = n(\mu_4(X_i) - 3\mu_2(X_i))$$

Here μ_4 is 4th central moment and μ_2 is 2nd central moment (variance).