Homework 6

MATH 281A

November 17, 2014

Exercise 3.18.

We use Method I to derive the contradiction. Pay special attention to the constant coefficient in both side of the polynomials.

Exercise 3.19.

This is a tricky one. Pay attention that the summation of the two truncated Poisson variables is NOT a truncated Poisson with parameter as the summation of the two parameters.

To attack the problem, go through these steps. We set $n = 1$. First calculate the mean of the truncated Poisson. One easy way to do this is using

$$\theta = (1 - \mathbb{P}(\tilde{X} = 0))\,\mathbb{E}(X)$$

where X follows truncated Poisson and $\tilde{X}$ follows Poisson with the same distribution θ. (Why we have this equation? Try to derive it.)

You may find that this equation does not really solve the question since $\mathbb{E}(X)$ is not a linear function of θ. One more ingredient is to consider $\mathbb{P}(X = 1)$ by using the following

$$\mathbb{P}(X = 1) = \frac{\mathbb{P}(\tilde{X} = 1)}{1 - \mathbb{P}(\tilde{X} = 0)}$$

where the setting is the same as before.

Finally try to construct θ using $\mathbb{E}(X)$ and $\mathbb{P}(X = 1)$. What is the statistic finally?

For $n = 2$, by a simple exponential family observation we can find sufficient and complete statistic as $X_1 + X_2$. Then suppose the UMVUE in part (a) is $T(X_1)$, we just need

$$\mathbb{E}(T(X_1)|X_1 + X_2)$$

You will encounter several problems when doing this Rao-Blackwellization stuff. If you get stuck, try augmenting the variable into full Poisson, then get the result and deduct the case when zero happens.

Good luck...

Exercise 20(C).

Direct calculation may yield the desired result. You only need to verify the C-R bound.

Exercise 2.1.

Follow the hint and you will be OK. Notice that σ^2 is a constant.

Exercise 2.5.

Use the standard move to show that the statistic is unbiased and based on the sufficient and complete statistic.

Exercise 2.9.

All you need to do is to show X sufficient and complete...

Exercise 2.15.

We may have done this question before.

Exercise 2.25.

This one is also not easy. First exhibit that $(X_{(n)}, Y_{(n)})$ is complete and sufficient (Easy). And intuitively we may look at statistic $X_{(n)}/Y_{(n)}$. To calculate the expectation of the statistic, the following lemma may be helpful.

For some nonnegative continuous random variable X, we have

$$\mathbb{E}(X) = \int_0^\infty \mathbb{P}(X > t)dt$$

Exercise 2.27.

Once we have (a), then (b) is obvious since $MSE = bias^2 + Variance$ and the variance of MLE is clearly smaller by the hint.

For (a), note that $\xi = u$ is equivalent to the fact that $p = \Phi(\xi - u) = 0.5$. Then we need to show that $\mathbb{E}\,\Phi(u - \bar{X}) = 0.5$. In fact you just need to show that $\Phi(u - \bar{X})$ follows uniform distribution on $(0, 1)$.